

# Emergent Geometry Fluctuation in Quantum Confined Electron Systems

Areg Ghazaryan and Tapash Chakraborty

*Department of Physics and Astronomy, University of Manitoba, Winnipeg, Canada R3T 2N2*

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The intrinsic geometric degree of freedom that was proposed to determine the optimal correlation energy of the fractional quantum Hall states, is analyzed for quantum confined planar electron systems. One major advantage in this case is that the role of various unimodular metrics resulting from the absence of rotational symmetry can be investigated independently or concurrently. For interacting electrons in our system, the confinement metric due to the anisotropy shifts the minimum of the ground state and the low-lying excited states from the isotropic case much more strongly than the corresponding shift due to the unimodular Galilean metric. Implications of these results for possible observation of higher Landau level filling fractions have been elucidated.

In a two-dimensional electron gas (2DEG) subjected to a strong perpendicular magnetic field, the ground state corresponding to the  $\frac{1}{3}$  filling factor of the lowest Landau level [1, 2] is described by the celebrated Laughlin wave function [3]. While investigating the origin of the success of that wave function, Haldane [4] recently realized that, contrary to the popular belief, there is a hidden geometrical fluctuation corresponding to the anisotropic correlation hole around each electron in the system. This anisotropy occurs either in the presence of an anisotropic interaction or the band mass anisotropy that gives rise to a unimodular (area preserving) Galilean metric [5]. In fact, the interaction metric is not necessarily congruent to the Galilean metric. Interestingly, the latter metric was shown to be equivalent [6] to what one obtains in the case of a magnetic field that is tilted [2, 7] from the direction perpendicular to the electron plane. Consequent to this theory of Haldane, there were a few numerical studies reported in the literature [8–11], that explored the influence of the Coulomb or Galilean metric corresponding to a particular anisotropic Hamiltonian. In these approaches the correlation metric was taken to interpolate [4] between the two other metrics. It provides the variational parameter one needs to minimize the correlation energy in the Laughlin state. The lowest Landau level fractional quantum Hall (FQH) states were found to be robust against variation of the anisotropy introduced through the intrinsic metric, while the FQH states at higher Landau levels are susceptible to compressible - incompressible phase transitions due to the anisotropy [6]. In the present work, we report on our study of the quantum-confined electron systems [12] where we introduce various unimodular metrics that can be varied independently or conjunctionally, thereby providing information about their influence on the energy spectra. Our results indicate that, in a single electron system the mass anisotropy and the confinement anisotropy will generate identical effects. In the case of interacting electrons in the system, the confinement anisotropy shifts the minimum of the ground state and the low-lying excited states from the isotropic case much more strongly than the corresponding shift due to the mass anisotropy alone. A suitable combination of these two unimodular metrics would perhaps generate a more pronounced FQHE in experi-

ments at higher Landau level filling factors.

We consider the two-dimensional electron gas subjected to a perpendicular magnetic field and in a parabolic confinement. The many-body Hamiltonian can then be written in the form

$$\mathcal{H} = \sum_i^N \mathcal{H}_i^e + \frac{1}{2} \sum_{i,j}^N V_{ij}, \quad (1)$$

where  $\mathcal{H}_i^e$  is a one electron Hamiltonian which with the inclusion of the effective mass and the confinement anisotropy is written as

$$\mathcal{H}_i^e = \frac{1}{2m_e} \left[ (\Pi_x)^2 / \alpha_\mu + \alpha_\mu (\Pi_y)^2 \right] + \frac{1}{2} m_e \omega_0^2 (\alpha_C x^2 + y^2 / \alpha_C) + \frac{1}{2} g \mu_B B \sigma_z. \quad (2)$$

Here  $\mathbf{\Pi} = \mathbf{p} - \frac{e}{c} \mathbf{A}$ ,  $\omega_0$  is the confinement potential strength for the isotropic case, while  $\alpha_\mu$  and  $\alpha_C$  are the mass and confinement anisotropy parameters respectively. We employ the symmetric gauge vector potential  $\mathbf{A} = \frac{1}{2}(-y, x, 0)B$ . The third term on the right hand side of Eq. (2) is the Zeeman energy. The second term in Eq. (1) is the Coulomb interaction which in the case of the anisotropic dielectric tensor has the form

$$V_{ij} = \frac{e^2}{\epsilon \sqrt{\alpha_I (x_i - x_j)^2 + (y_i - y_j)^2 / \alpha_I}}, \quad (3)$$

where  $\alpha_I$  is the interaction anisotropy parameter, and the directions of  $\hat{x}$  and  $\hat{y}$  are along the principal axes of the dielectric tensor. We first decouple the one-particle Hamiltonian into three parts

$$\begin{aligned} \mathcal{H}_A &= \frac{1}{2m_e \alpha_\mu} (p_x^2 + \Omega_x^2 x^2) + \frac{\alpha_\mu}{2m_e} (p_y^2 + \Omega_y^2 y^2), \\ \mathcal{H}_Z &= \frac{1}{2} g \mu_B B \sigma_z, \\ \mathcal{H}_R &= \frac{1}{2} \omega_c (\alpha_\mu x p_y - y p_x / \alpha_\mu), \end{aligned}$$

where  $\mathcal{H}_A$  describes the two-dimensional spinless harmonic oscillator with mass anisotropy in  $\hat{x}$  and  $\hat{y}$  directions. We have introduced the cyclotron frequency  $\omega_c = |e|B/m_e c$  and the oscillator frequencies  $\Omega_x^2 =$

$m_e^2 \alpha_\mu^2 (\frac{\alpha_C}{\alpha_\mu} \omega_0^2 + \frac{1}{4} \omega_c^2)$ ,  $\Omega_y^2 = \frac{m_e^2}{\alpha_\mu^2} (\frac{\alpha_\mu}{\alpha_C} \omega_0^2 + \frac{1}{4} \omega_c^2)$ . The eigenstates  $|\lambda\rangle$  of  $\mathcal{H}_\Lambda$  are the direct products  $|n_x^\lambda\rangle|n_y^\lambda\rangle$  of two harmonic oscillator states represented by the quantum numbers  $n_{x,y}^\lambda$ . Inclusion of the Zeeman term  $\mathcal{H}_Z$  is done by multiplying the states  $|\lambda\rangle$  by the eigenstates of the Pauli spin matrix  $\sigma_z$ . Finally, the  $\mathcal{H}_R$  part of the Hamiltonian mixes the states with different quantum numbers  $n_{x,y}^\lambda$  and therefore the spectra of the single-electron Hamiltonian should be obtained by employing the diagonalization procedure using the eigenstates of  $\mathcal{H}_\Lambda$  and  $\mathcal{H}_Z$  as the basis.

In order to evaluate the energy spectrum of the confined electron system we need to diagonalize the matrix of the Hamiltonian in Eq. (1) in a basis of the Slater determinants constructed from the single-electron eigenstates of the Hamiltonian in Eq. (2). To calculate the two-body Coulomb interaction matrix elements we use the procedure of Fourier transformation outlined previously [13]. The Fourier transform of the anisotropic interaction is also anisotropic and has the form

$$\tilde{V}_C(\mathbf{k}) = \frac{2\pi e^2}{\varepsilon \sqrt{k_x^2/\alpha_I + k_y^2\alpha_I}}. \quad (4)$$

Our numerical studies were carried out for the two-dimensional InAs system using the following parameters:  $m_e = 0.042m_0$ ,  $g_e = -14$ ,  $\varepsilon = 14.6$ ,  $\omega_0 = 4$  meV. We have considered the variation of all anisotropy parameters  $\alpha_\mu$ ,  $\alpha_C$  and  $\alpha_I$  in the range of 0.2 – 2. The value of  $\alpha_\mu = \alpha_C = \alpha_I = 1$  correspond to the isotropic case. The results for the more popular GaAs system are qualitatively similar to the present case.

Let us first discuss about the one electron case. In Fig. 1 the dependence of low-lying energy levels of a one-electron system on the mass anisotropy parameter  $\alpha_\mu$  is shown for various values of the magnetic field strength  $B$ . The confinement parameter is taken to be  $\alpha_C = 1$  (isotropic confinement potential). The spectrum is clearly symmetric under the transformation  $\alpha_\mu \rightarrow 1/\alpha_\mu$  and the minimum for the ground state appears in the isotropic case. Inclusion of the mass anisotropy brakes the rotational symmetry of the system which results in the lifting of the degeneracies for the excited states in the case of  $B = 0$ . It should be noted that the dependence of the energy levels on the confinement parameter  $\alpha_C$  for the case of  $\alpha_\mu = 1$  is exactly the same as in Fig. 1. This can be explained directly from the one-electron Hamiltonian of Eq. (2), by making a rescaling of the coordinates  $x \rightarrow x/\sqrt{\alpha_\mu}$ ,  $y \rightarrow y\sqrt{\alpha_\mu}$ . In that case the mass anisotropy will be transferred to the confinement anisotropy, with the direction of the hard and easy axes interchanged as compared to the anisotropy introduced by  $\alpha_C$ . The interchange of the hard and easy axes has no effect on the energy spectrum of the system. Therefore for the one-electron case it makes no difference whether the symmetry of the system is broken by inducing the mass anisotropy or the confinement anisotropy.

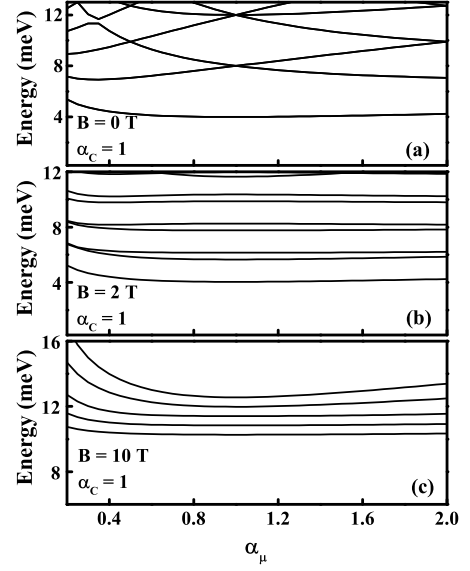


FIG. 1: Dependence of the low-lying energy levels of a single electron system on the mass anisotropy parameter  $\alpha_\mu$  for various values of the magnetic field strength  $B$ . The confinement is taken to be isotropic. For  $B = 0$  all states are Kramer doublets.

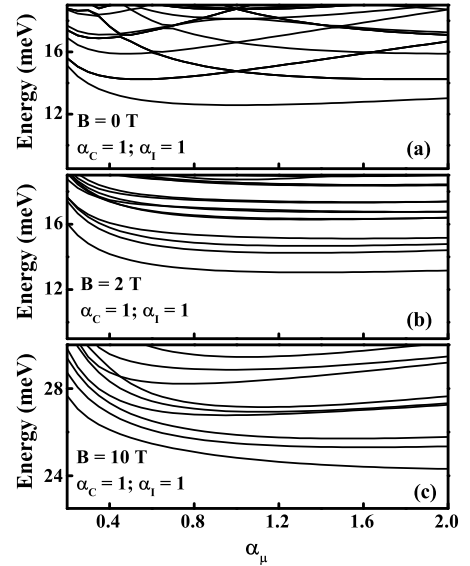


FIG. 2: Dependence of the low-lying energy levels of a two-electron system on the mass anisotropy parameter  $\alpha_\mu$  for various values of the magnetic field strength  $B$ . The confinement and the Coulomb interaction are taken to be isotropic.

This might have important consequences in actual experiments involving anisotropic electron systems.

In Fig. 2 we present the magnetic field dependence of the low-lying energy levels of a two-electron system on the mass anisotropy parameter  $\alpha_\mu$ . The confinement and the Coulomb interaction are considered to be isotropic ( $\alpha_C = 1$  and  $\alpha_I = 1$ ). In the absence of an external magnetic field, this anisotropy parameter dependence

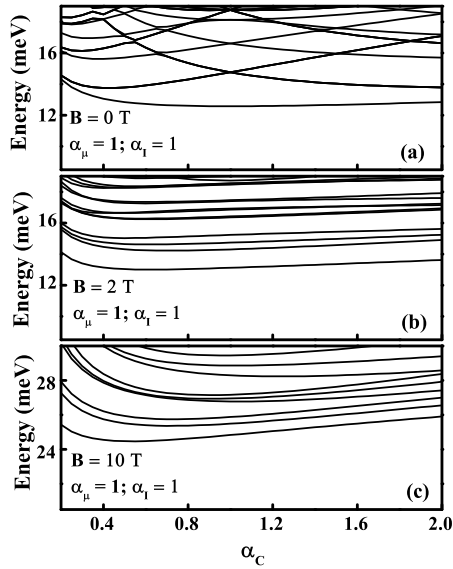


FIG. 3: Dependence of the low-lying energy levels of a two-electron system on the confinement anisotropy parameter  $\alpha_C$  for various values of the magnetic field strength  $B$ . The mass and the Coulomb interaction are taken to be isotropic.

strongly resembles that of the one-electron case. However, for the case of  $B = 2\text{T}$  or  $10\text{T}$ , the minimum of the ground state and the excited states are clearly shifted to higher values of the anisotropy parameter, and the case of  $\alpha_\mu = 1$  does not correspond to a special point. It should be pointed out that, interestingly a similar behavior was reported earlier for the FQHE state with filling factor  $\nu = 1 + 1/3$  [6] from theoretical studies of 8 electrons in a periodic rectangular geometry. We have shown previously [14, 15] that in an anisotropic system and for the one-electron case the effect of the magnetic field is to rotate the directions of the oscillator motion from  $\hat{x}$  and  $\hat{y}$  directions. By making the same rescaling as for the one-electron case mentioned above, the inclusion of mass anisotropy (or any one anisotropy) for the many-electron case can be transferred to the anisotropies of the parabolic confinement and the Coulomb interaction (or to the other two anisotropies), both having the same hard axis, as long as the metrics are diagonal. The change in anisotropy of the confinement potential also affects the angle of rotation of the oscillator motion induced by the magnetic field. Hence the shift of the minimum of the ground and excited states to higher anisotropy values is an interplay between the rotated oscillator motion which is determined by the magnetic field strength, the confinement anisotropy, and the Coulomb interaction anisotropy.

In Fig. 3, we present the dependence of the low-lying energy levels of a two-electron system on the confinement anisotropy parameter  $\alpha_C$  for various values of the magnetic field strength  $B$ . The mass and the Coulomb interaction are taken to be isotropic ( $\alpha_\mu = 1$  and  $\alpha_I = 1$ ). Here we notice that, again for the case of the non-zero

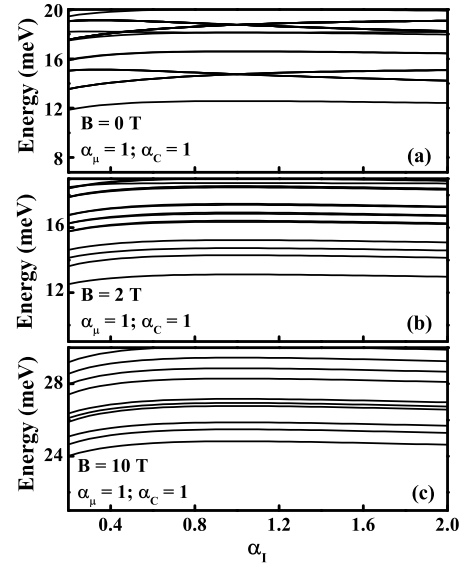


FIG. 4: Dependence of the low-lying energy levels of a two-electron system on the interaction anisotropy parameter  $\alpha_I$  for various values of the magnetic field strength  $B$ . The mass and the confinement are taken to be isotropic.

magnetic field the minimum of the ground and the excited states are shifted to a lower value of the corresponding anisotropy parameter. It should be pointed out that in this case the shifting of the minimum is more pronounced than in the case of the mass anisotropy. Hence the sole inclusion of the confinement anisotropy has a more profound effect on the energy levels of the system than the inclusion of the mass anisotropy alone. The implication of this results for experimental observation of the FQHE states will be elaborated below.

Finally, we consider the case of the interaction anisotropy. The dependence of the low-lying energy levels of a two-electron system on the interaction anisotropy parameter  $\alpha_I$  for various values of the magnetic field strength  $B$  is presented in Fig. 4. The mass and the confinement potential are considered here to be isotropic ( $\alpha_\mu = 1$  and  $\alpha_C = 1$ ). Here we notice that the energy levels have the maximum at the isotropy point for all values of the magnetic field. Due to the isotropy of the mass and the confinement potential, the one-electron wave function possesses the rotational symmetry. As a consequence, the Coulomb interaction is more pronounced when it is symmetric and the electrons interact equivalently in all directions. We can then conclude that the isotropic interaction case should have the maximum energy compared to the case of the anisotropic interaction.

If the band mass anisotropy is inherent in the semiconductor system considered here and not induced by an external means, such as the tilted magnetic field, one may argue that the same mass anisotropy should be applied also to the confinement potential strength parameter, where the electron effective mass also appears. In fact, systems with internal band mass anisotropy

were suggested by the recent experiments on AIAs two-dimensional electron systems [16]. For the one-electron system if both the kinetic term and the parabolic confinement in the Hamiltonian in Eq. (2) possess the same anisotropy, then using the coordinate rescaling mentioned above it can be shown that these two metrics are congruent, so that the rotational symmetry is preserved. As for two-electron case using the same coordinate rescaling in the Hamiltonian (1) this anisotropy can be transferred to the anisotropy of the Coulomb interaction. If any other anisotropy is imposed on the confinement potential by an external means, the anisotropy of the Coulomb interaction will surely move the minimum of the dependence of energy levels on the additional confinement anisotropy parameter even farther away from the isotropic case, due to the maximum observed for the Coulomb interaction anisotropy in the isotropic case.

Experimentally, the tilt-induced anisotropy in the FQHE states have been well studied in recent literature [17, 18] in addition to its earlier success in exploring spin polarizations [19, 20] in the lowest Landau level [7]. In a tilted magnetic field a stable FQHE state for the filling factor  $\nu = \frac{7}{3} (= 2 + \frac{1}{3})$  has been recently reported [18]. These authors pointed out that albeit the presence of the quantized Hall plateau, the longitudinal resistance possesses strong temperature-dependent anisotropy. Theoretically, it was shown that the effective mass tensor of the 2DEG can be tuned by the tilted magnetic field [6], and therefore the observation of this special phase at  $\nu = 7/3$  can be related to the FQHE with mass anisotropy. It should be pointed out that the tilted magnetic field couples the planar motion of the electrons in a 2DEG with the perpendicular motion, rendering the electron dynamics in the FQHE states highly

non-trivial. Perhaps the mass anisotropy (or as suggested above, a combination of the mass anisotropy and the confinement anisotropy) will provide a better route to study anisotropic states in the FQHE experiments.

In summary, we have shown that for a two-electron system in a parabolic confinement under the influence of a magnetic field, the confinement anisotropy has the similar effect on the system as does the mass anisotropy. We have also pointed out that in the case of the confinement anisotropy the shifting of the minimum of the ground state and low-lying excited states to lower values of the corresponding anisotropy parameter is more pronounced than that for the mass anisotropy. As we have mentioned above, a similar kind of shifting of the minimum from the isotropic case was also reported theoretically for the energy spectrum of 2DEG at filling factor  $\nu = 1 + 1/3$ . Although these two calculations are not directly related, the similarity of this shift of the minimum helps us make a prediction that in the FQHE experiments if one introduces a confinement potential in the 2DEG, the anisotropy of that confinement potential will have a similar dramatic effect as that of the tilted magnetic field. We therefore believe that the simultaneous use of both the tilted magnetic field and the confinement anisotropy would strongly enhance the resultant anisotropy than the tilt-induced anisotropy alone in the system. That would most likely make the observation of the anisotropic FQHE phases even more pronounced. A similar study of larger electron systems will be the subject of our future publications.

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